Thermodynamics of SU(4) gauge theory with fermions in multiple representations

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“Multirep” Theories

General class of theories:

One gauge field

Multiple fermion fields coupled to the gauge field

Fermion fields charged under different irreps of the gauge group

This study (“Our theory”):

SU(4) gauge theory

$\boxed{F}$

$N_F = 2$ Dirac fermions in fundamental irrep of SU(4) $[F\ q_i\ \mathbf{4}\ \phi]$

$N_{A_2} = 2$ Dirac fermions in 2-index antisymmetric irrep of SU(4) $[A_2\ \mathrm{AS2}\ Q_i\ \mathbf{6}\ \theta]$

$\begin{array}{|c|}\hline A_2 \\ \hline \\ \hline\end{array}$

Motivation & Outline

Separated Phases: Can gauge theories exhibit "partially confined" phases between the usual "confined & chirally broken" and "deconfined & chirally restored"?

Multirep theories have many (conceptually) distinct transitions

- Confinement and chiral transitions for each irrep
- E.g., our theory has 4: chiral F, chiral A_2, confinement F, confinement A_2
- All occur simultaneously, or are they separated?

Order of Transition(s): What is the order of transition(s) encountered?

First-order transitions in the early universe produce gravitational waves

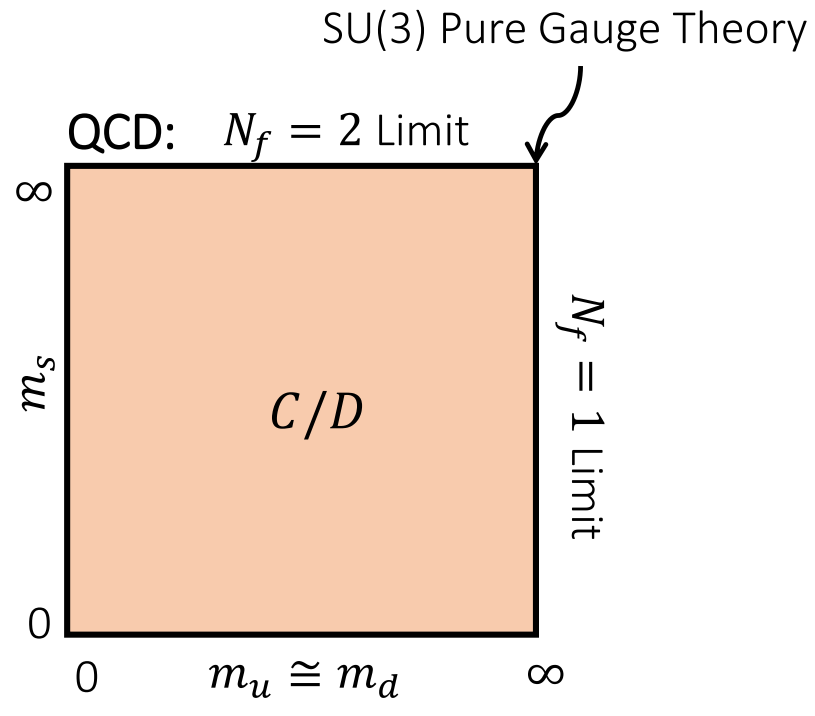
- May be measurable by near future detectors [arXiv:1512.06239]
- Probe of dark sectors with small couplings to Standard Model [arxiv:1504.07263]

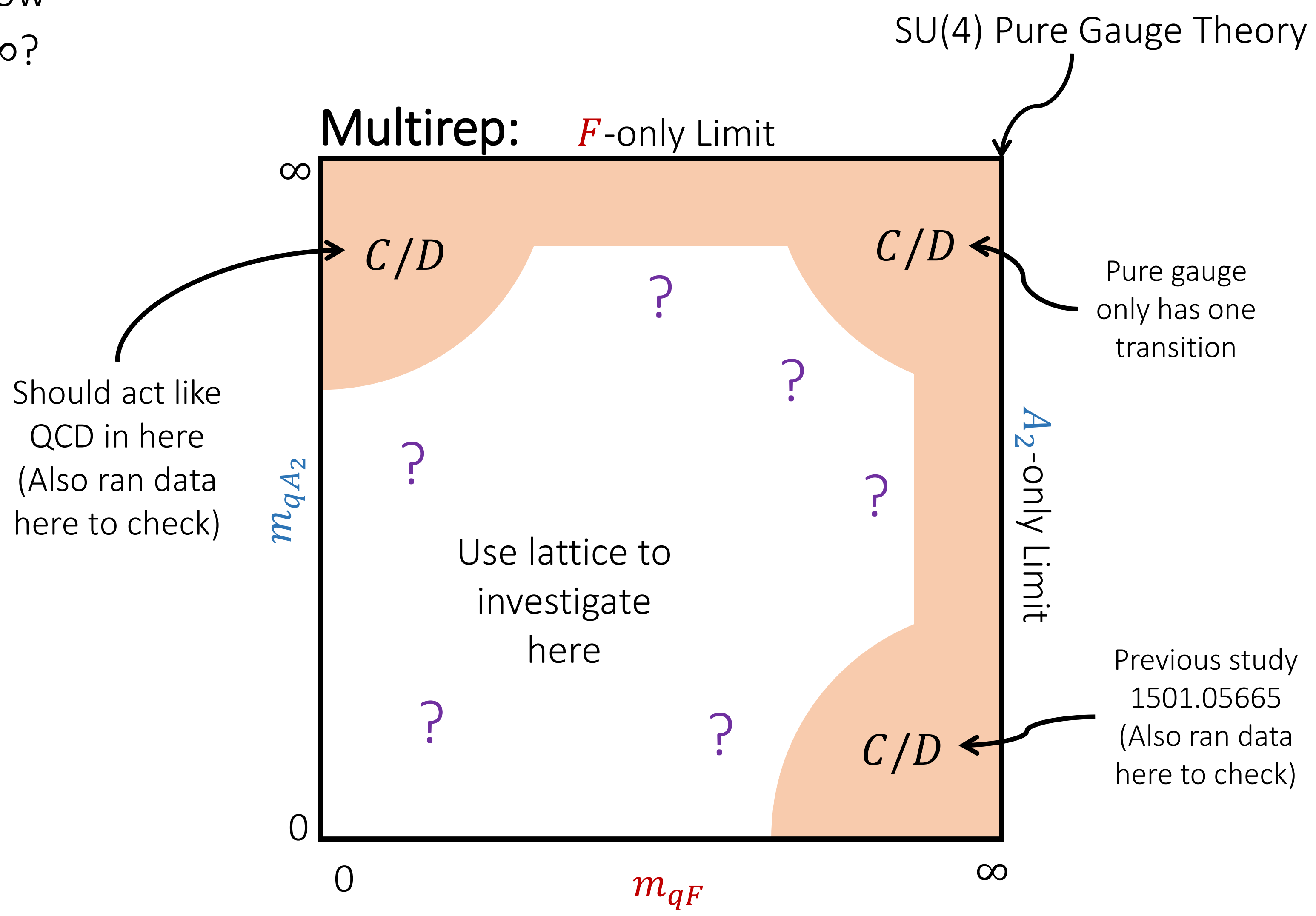
Analytic result: "Multirep Pisarski-Wilczek"

Separated Phase "Columbia Plot"

Given some fixed quark masses m_F, m_{A_2}, how many transitions between $T = 0$ and $T = \infty$?

$C/D \equiv$ "Only one transition is present, between Confined and Deconfined phases"





MAC Hypothesis & Predictions

One-gluon exchange ⇒ Potential between quarks scales like Casimirs [RDS 1980]

$$V \sim g^2(\mu)\left[C_c^2 - C_{r_1}^2 - C_{r_2}^2\right]$$

As scale runs down, most attractive channel (MAC) chirally condenses first

For our theory, possible channels:

$$V_{F,\bar{F}\to 1} \sim -15/2$$

$$\boxed{V_{A_2,A_2\to 1} \sim -10} \longleftarrow \text{MAC}$$

$$V_{F,A_2\to \bar{F}} \sim -5$$

$$V_{F,A_2\to 20} \sim +1$$

(Naïve) MAC expectation: $A_2, A_2 \to 1$ first, then $F, F \to 1$

Lattice Result: All Transitions Coincide

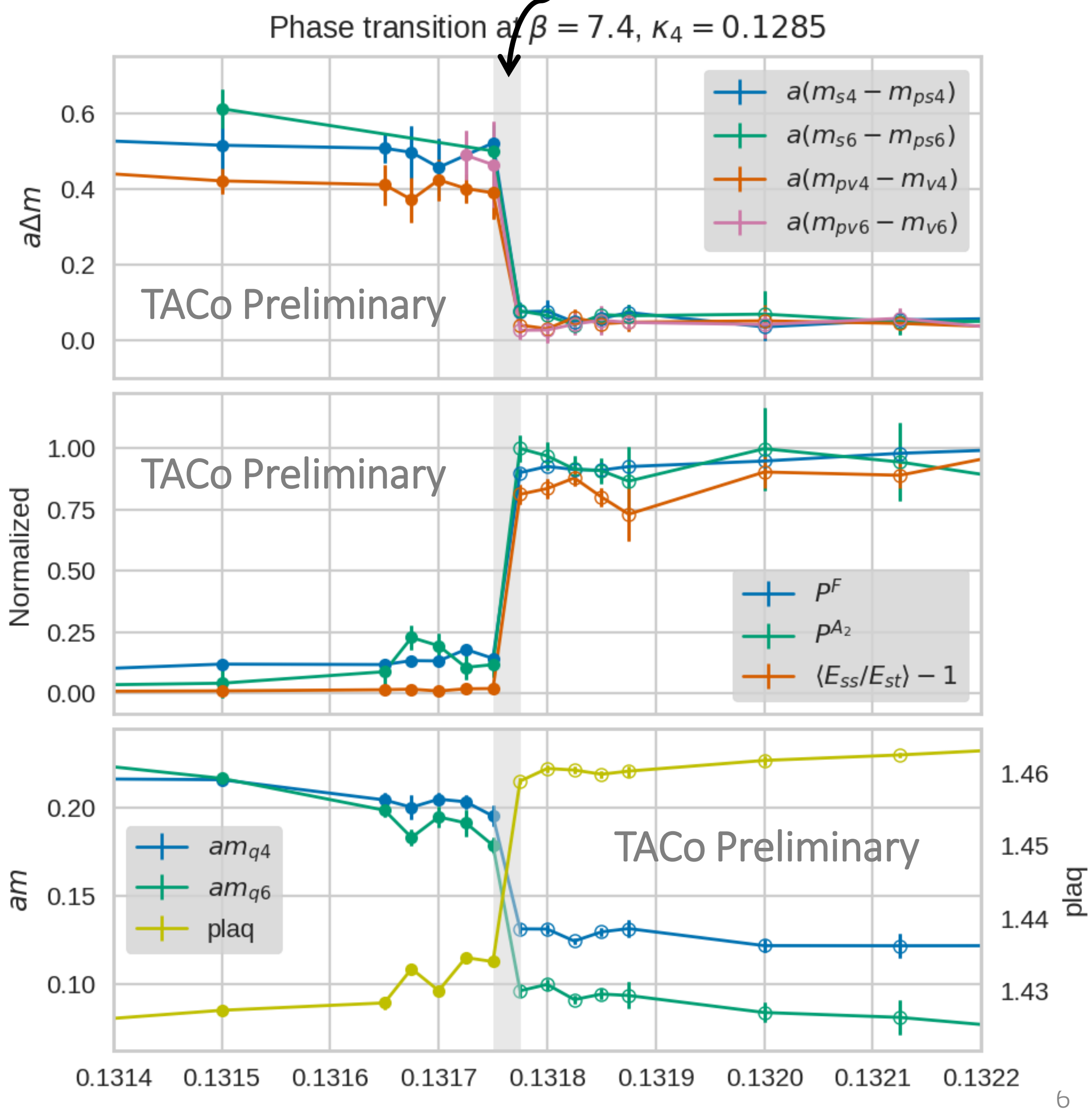
Gray Band: Location of transition

Chiral Transition Diagnostics

(Parity doubling: In chirally restored phase, scalar and pseudoscalar become degenerate)

(Normalized) Confinement Diagnostics

(Polyakov loop for each irrep of fermion is small when those fermions are confined, large when deconfined)



All species of transition are simultaneous

Typical slice for explored regions of parameter space

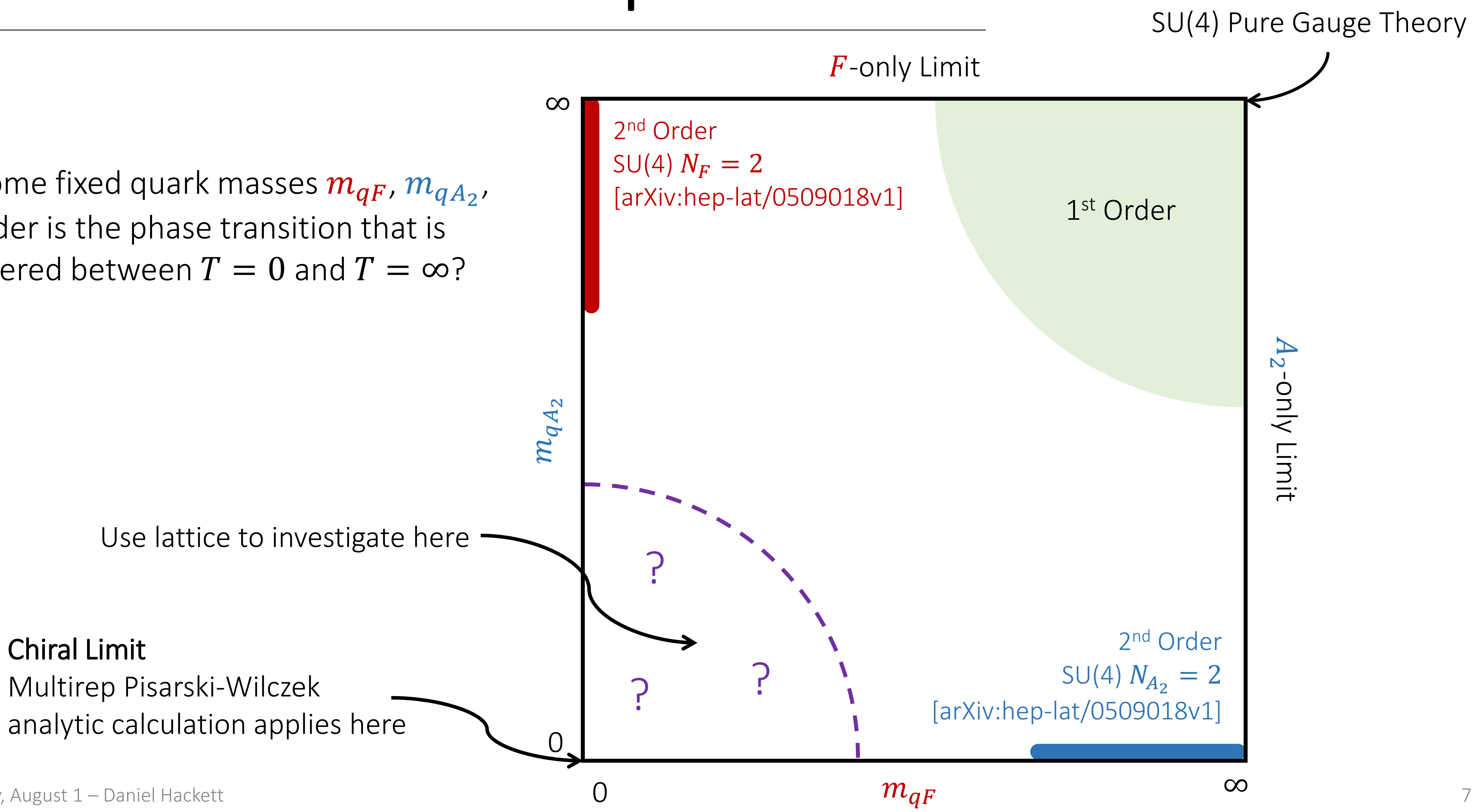
We have not observed phase separation.

Closed circles: Confined ensembles

Open circles: Deconfined ensembles

Transition Order: Multirep Columbia Plot

Given some fixed quark masses m_{qF}, m_{qA_2}, what order is the phase transition that is encountered between $T = 0$ and $T = \infty$?



Multirep Pisarski-Wilczek

Examine critical behavior of EFT of bilinear quark condensates

- $\Longleftrightarrow$ Analyze fixed points of a linear sigma model

Procedure [Pisarski & Wilczek 1984; arXiv:hep-lat/0509018v1]:

- Identify symmetries and SSB pattern
  - Work in chiral limit, $m_F = m_{A_2} = 0$
- Write down most general Lagrangian consistent with symmetries
  - Only include relevant and marginal terms
- Compute β functions (to one loop)
  - We only observe one transition, so drive both irreps to criticality simultaneously
  - ϵ expansion: At finite T, theory becomes effectively 3D, so set $\epsilon = 4 - d = 1$
- Perform stability analysis
  - No IR-stable fixed points exist $\Rightarrow$ transition **must** be first order
  - Any IR-stable fixed points exist $\Rightarrow$ transition **can** be second-order

Multirep P-W: Symmetries

Fermion irreps have independent axial rotations: $\mathrm{U}(1)_A^{(F)}$, $\mathrm{U}(1)_A^{(A_2)}$

There is only one axial anomaly ⇒ There is a non-anomalous $\mathrm{U}(1)_A$ symmetry!

In our theory, the non-anomalous $\mathrm{U}(1)_A$ obeys the condition:

$$\alpha_F = 2\alpha_{A_2}$$

[Clark, Leung, Love, Rosner 1986; DeGrand, Golterman, Neil, Shamir 2016]

where

$$q \to e^{i\alpha_F \gamma_5} q$$
$$Q \to e^{i\alpha_{A_2} \gamma_5} Q$$

[Behavior under axial rotations]

Full SSB pattern (A_2 is a real irrep, so $\mathrm{SU}(2N_{A_2}) \to \mathrm{SO}(2N_{A_2})$)

$$\mathrm{SU}(N_F)_L \times \mathrm{SU}(N_F)_R \times \mathrm{SU}(2N_{A_2}) \times \mathrm{U}(1)_A \to \mathrm{SU}(N_F)_V \times \mathrm{SO}(2N_{A_2})$$

Induce SSB by tuning potential ⇒ LHS must be good symmetry of Lagrangian

Multirep P-W: Results

Write down Lagrangian

Discover that all terms that "implement the anomaly" but respect $\alpha_F = 2\alpha_{A_2}$ are irrelevant

$\Rightarrow$ Both $\mathrm{U}(1)_A^{(F)}$, $\mathrm{U}(1)_A^{(A_2)}$ are good symmetries of the (relevant) Lagrangian

$\Rightarrow$ Anomaly does not play a role in dynamics of the phase transition!

Turn the crank (Complex ϕ^4 + group theory, some Mathematica…)

Find 6 fixed points (FPs)

(Only 2 are novel, 4 are decoupled products of FPs from F-only and A_2-only theories)

At one loop in the ϵ expansion, none of these fixed points are stable.

$\Rightarrow$ Prediction: transition must be first order!

Lattice Results: Transition is First Order

Gray Band: Location of transition

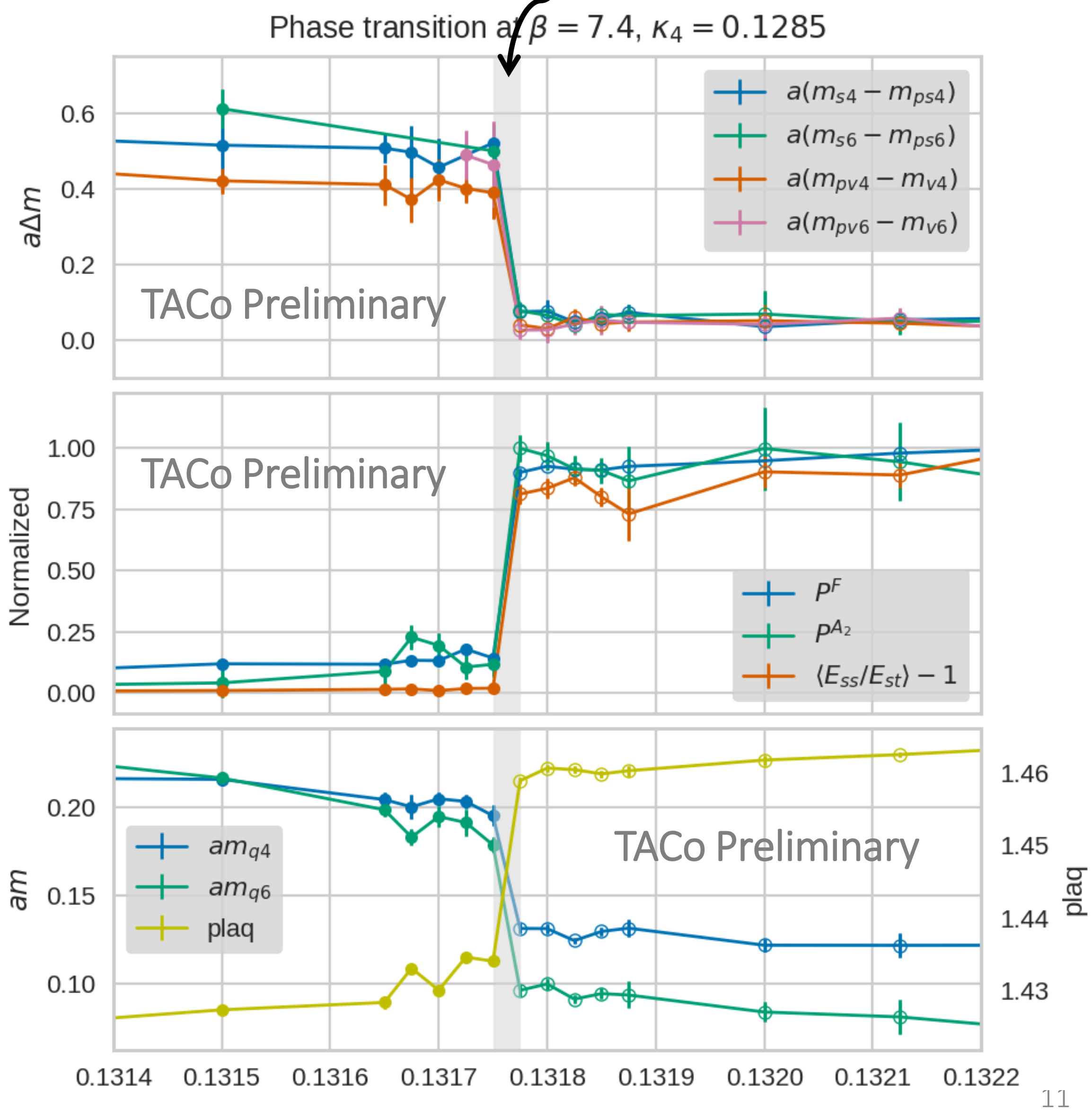
Every observable we've looked at jumps discontinuously at transition

Transition is sharp

- Interpolating more densely does not smooth out the transition
- Observables are either "confined-like" or "deconfined-like," with no interpolation

Discontinuity is present everywhere that we've looked in parameter space

⇒ (Violently) first-order transition!



Left axis: Quark masses
(Axial Ward Identity/PCAC, in lattice units)
Right axis: Plaquette expectation value

Conclusion

Did not observe phase separation

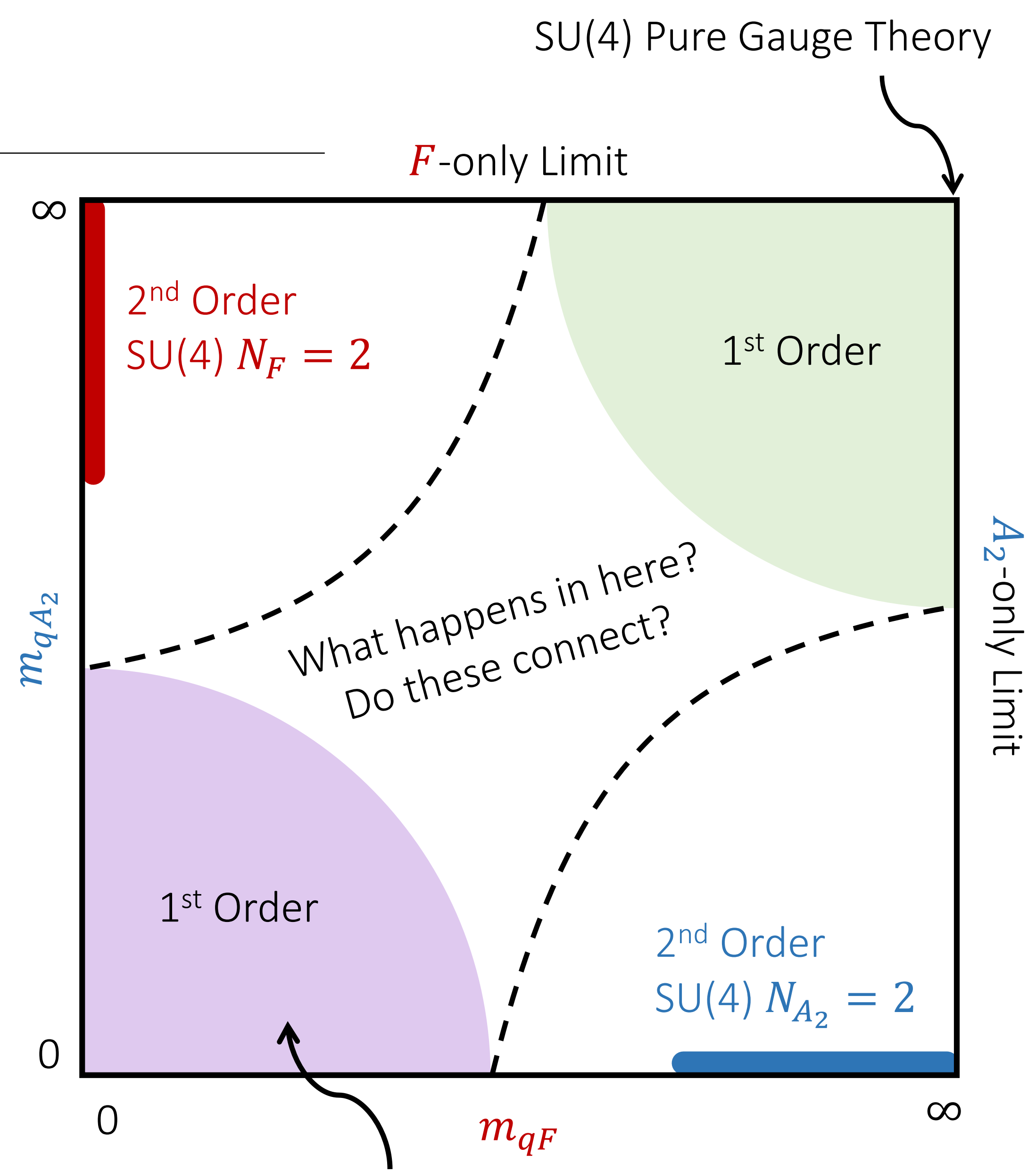
We have not explored parameter space exhaustively, but it's certainly not a typical or ubiquitous feature of the dynamics.

Phase structure looks like QCD

Transition is first order

Multirep Pisarski-Wilczek calculation suggests that it must be

Open question: First order all the way from pure gauge limit to multirep chiral limit?



Used lattice to investigate here
(First-ever study of multirep thermodynamics!)

Software/Numerical Methods

Multirep MILC [Shamir]

- SU(4) gauge theory in 3+1 dimensions
- nHYP smearing
- Clover-improved Wilson fermions (This study: $c_{sw} = 1$)
  - (Can run arbitrary number of simultaneous species in F, A_2, S_2, G!)
- nHYP Dislocation Suppressing (NDS) action [DeGrand, Shamir, Svetitsky 2014]

Spectroscopy

- Screening masses
- “Periodic+Antiperiodic BCs” trick enables spectroscopy on small lattices

Dataset

3D bare parameter space $(\beta, \kappa_F, \kappa_{A_2})$

Look at data along densely interpolated slice across the transition (boxed data in plot)

Lattices are $12^3 \times 6$

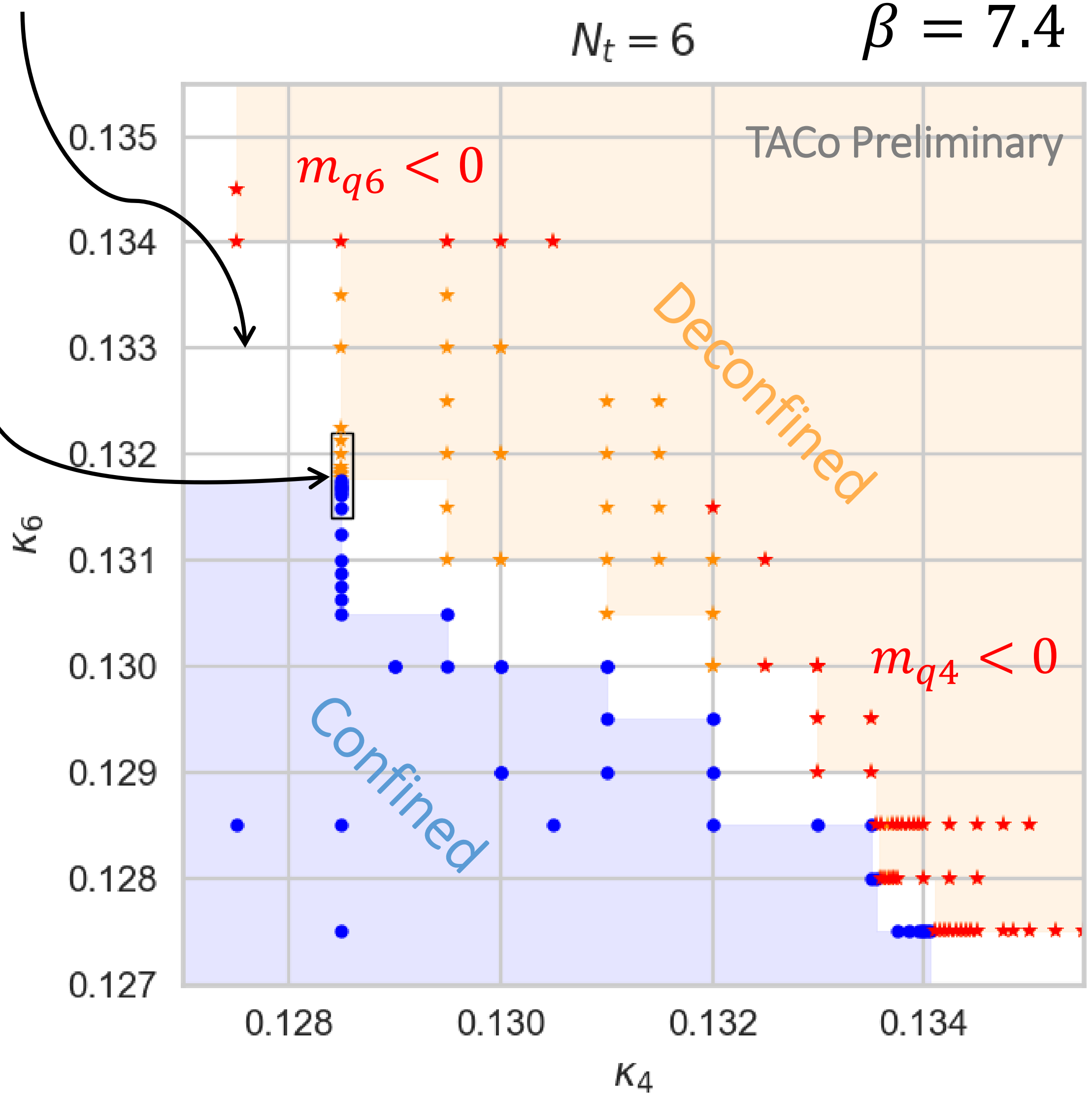
Have also looked at larger volumes, but have best interpolation on $12^3 \times 6$

Typically 30-200 decorrelated configs/ensemble

Fit spatial-direction correlators to get screening masses

- Confined ensemble, $m_q > 0$ (both reps)
- Confined ensemble, $m_q < 0$ (one or both reps)
- Deconfined ensemble, $m_q > 0$ (both reps)
- Deconfined ensemble, $m_q > 0$ (one or both reps)

White band: phase is ambiguous
(Transition lives in here)



Chiral Phase Diagnostic: Parity Doubling

Diagnose condensation by comparing masses of parity partners:

Chirally restored:

$m_{\mathrm{s}} - m_{\mathrm{ps}} = 0$ [Scalar and pseudoscalar are degenerate]

$m_{\mathrm{pv}} - m_{\mathrm{v}} = 0$ [Pseudovector and vector are degenerate]

Chirally broken:

$m_{\mathrm{s}} - m_{\mathrm{ps}} > 0$ [Scalar and pseudoscalar are non-degenerate]

$m_{\mathrm{pv}} - m_{\mathrm{v}} > 0$ [Pseudovector and vector are non-degenerate]

Do observables for each irrep respond simultaneously or separately?

(**Note:** Wilson fermions explicitly break chiral symmetry, so degeneracy in chirally restored phase is only approximate – but, small effect in this study.)

Multirep P-W: Lagrangian

Field content and required symmetries of the Lagrangian

$\phi \to e^{2i\alpha_F} U_L \phi U_R$ [$N_F \times N_F$ complex matrix field for F sector]

$\theta \to e^{2i\alpha_{A_2}} V \theta V^T$ [$2N_{A_2} \times 2\, N_{A_2}$ symmetric complex matrix field for A_2 sector]

No relevant anomaly terms

$\det\phi \det\theta^\dagger + c.c.$ respects $\alpha_F = 2\alpha_{A_2}$, but is dimension 6

$\Rightarrow$ Both $\mathrm{U}(1)_A^{1(F)}$ and $\mathrm{U}(1)_A^{(A_2)}$ are good symmetries separately.

Full relevant Lagrangian

$$\mathcal{L} = \mathrm{Tr}\left[\partial_\mu \phi^\dagger \partial^\mu \phi\right] + r_F \mathrm{Tr}\left[\phi^\dagger \phi\right] + v_F \left(\mathrm{Tr}\left[\phi^\dagger\phi\right]\right)^2 + u_F \mathrm{Tr}\left[\left(\phi^\dagger\phi\right)^2\right]$$
$$+ \mathrm{Tr}\left[\partial_\mu \theta^\dagger \partial^\mu \theta\right] + r_{A_2} \mathrm{Tr}\left[\theta^\dagger \theta\right] + v_{A_2} \left(\mathrm{Tr}\left[\theta^\dagger\theta\right]\right)^2 + u_{A_2} \mathrm{Tr}\left[\left(\theta^\dagger\theta\right)^2\right]$$
$$+ w \mathrm{Tr}\left[\phi^\dagger\phi\right]\mathrm{Tr}\left[\theta^\dagger\theta\right]$$

[Only relevant irrep-coupling term]

Multirep P-W: Beta functions

$$\beta_{u_F} = -u_F + (N_F^2 + 4)u_F^2 + 4N_F v_F u_F + 3v_F^2 + 2N_{A_2}^w\left(N_{A_2}^w + 1\right)w^2$$

$$\beta_{v_F} = -v_F + 2N_F v_F^2 + 6v_F u_F$$

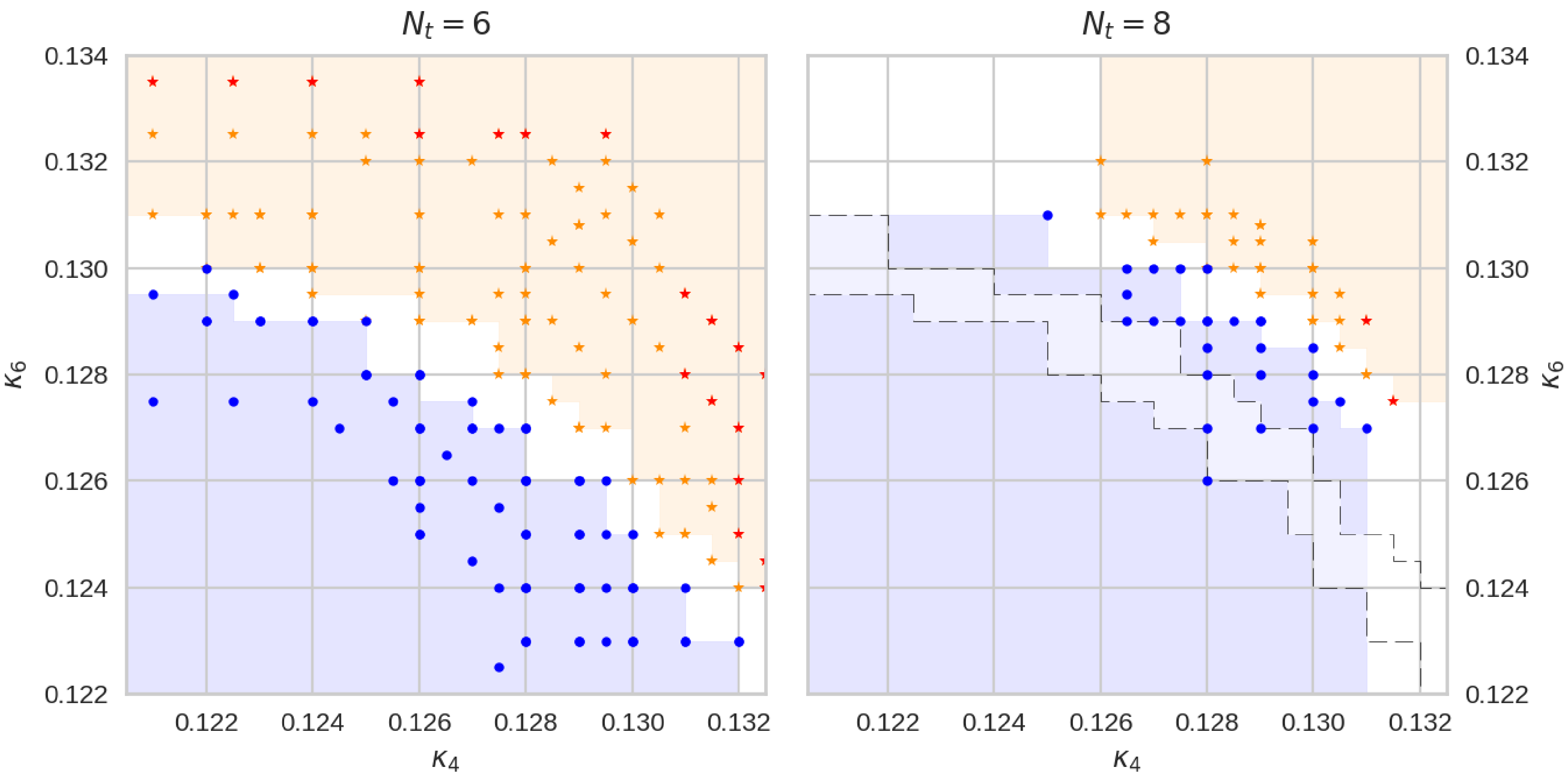
$$\beta_{u_{A_2}} = -u_{A_2} + \frac{1}{2}\left({N_{A_2}^w}^2 + N_{A_2}^w + 8\right)u_{A_2} + 2\left(N_{A_2}^w + 1\right)u_{A_2}v_{A_2} + \frac{3}{2}v_{A_2}^2 + 4N_F^2 w^2$$

$$\beta_{v_{A_2}} = -v_{A_2} + \left(N_{A_2}^w + \frac{5}{2}\right)v_{A_2}^2 + 6v_{A_2}u_{A_2}$$

$$\beta_w = -w + w\left(2N_F v_F + (N_F^2 + 1)u_F + \left(N_{A_2}^w + 1\right)v_{A_2} + \frac{1}{2}\left({N_{A_2}^w}^2 + N_{A_2}^w + 4\right)u_{A_2} + 2w\right)$$

Transition is Not a Bulk Transition

Fixed $\beta = 7.75$, explore (κ_4, κ_6) phase diagram. Varying the temporal extent N_t, the transition moves.



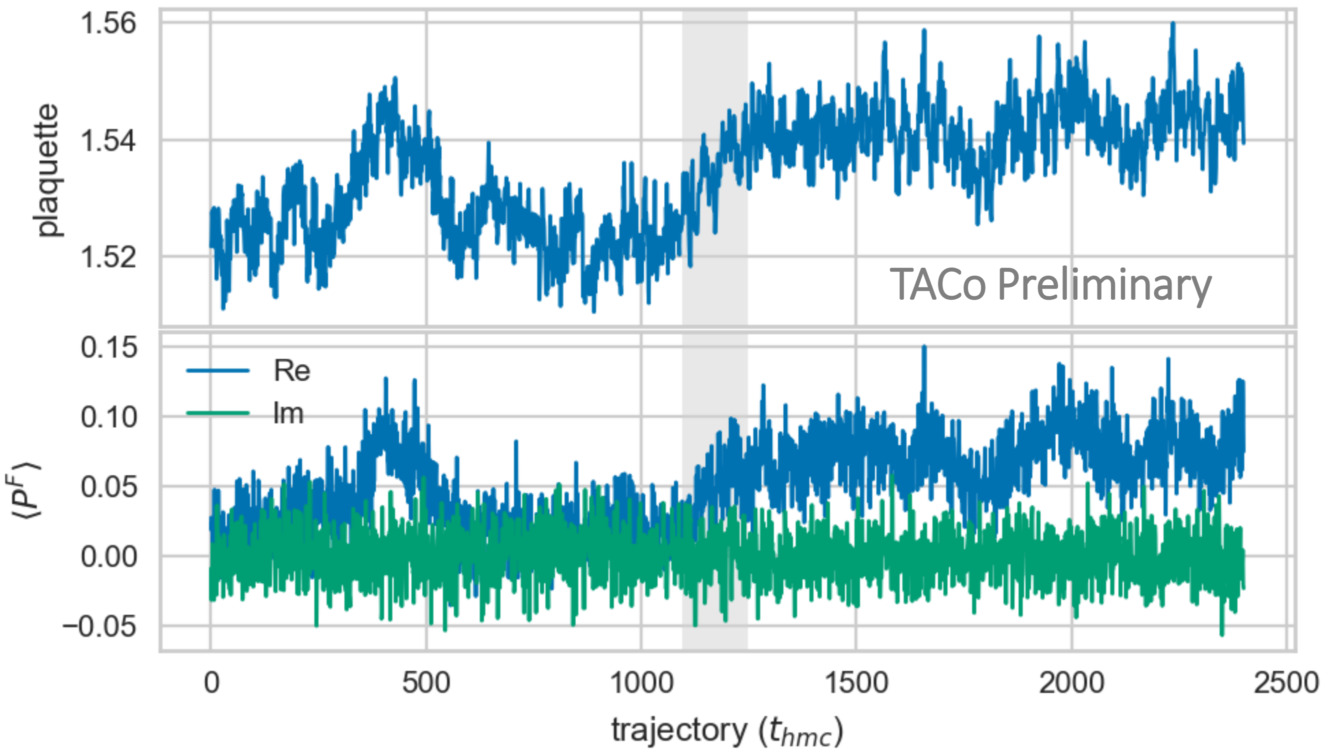
Transition is First Order: Metastability

Plot: Metastability in HMC time

- Ensemble seeded from a confined lattice with parameters close to transition
- Ran ensemble at deconfined parameters close to transition
- Plaquette wanders until "locking in" around trajectory ~ 1200
- Appears confined until trajectory ~ 1200, when it tunnels to deconfined state
- Spectroscopy observables also changed from "confined-like" to "deconfined-like" after tunneling
- **Typical equilibration time for $12^3 \times 6$ lattices is less than 100 trajectories**

Metastability is another signal of a first-order transition (cf. hysteresis)!

We have observed several tunneling events in the course of running our data



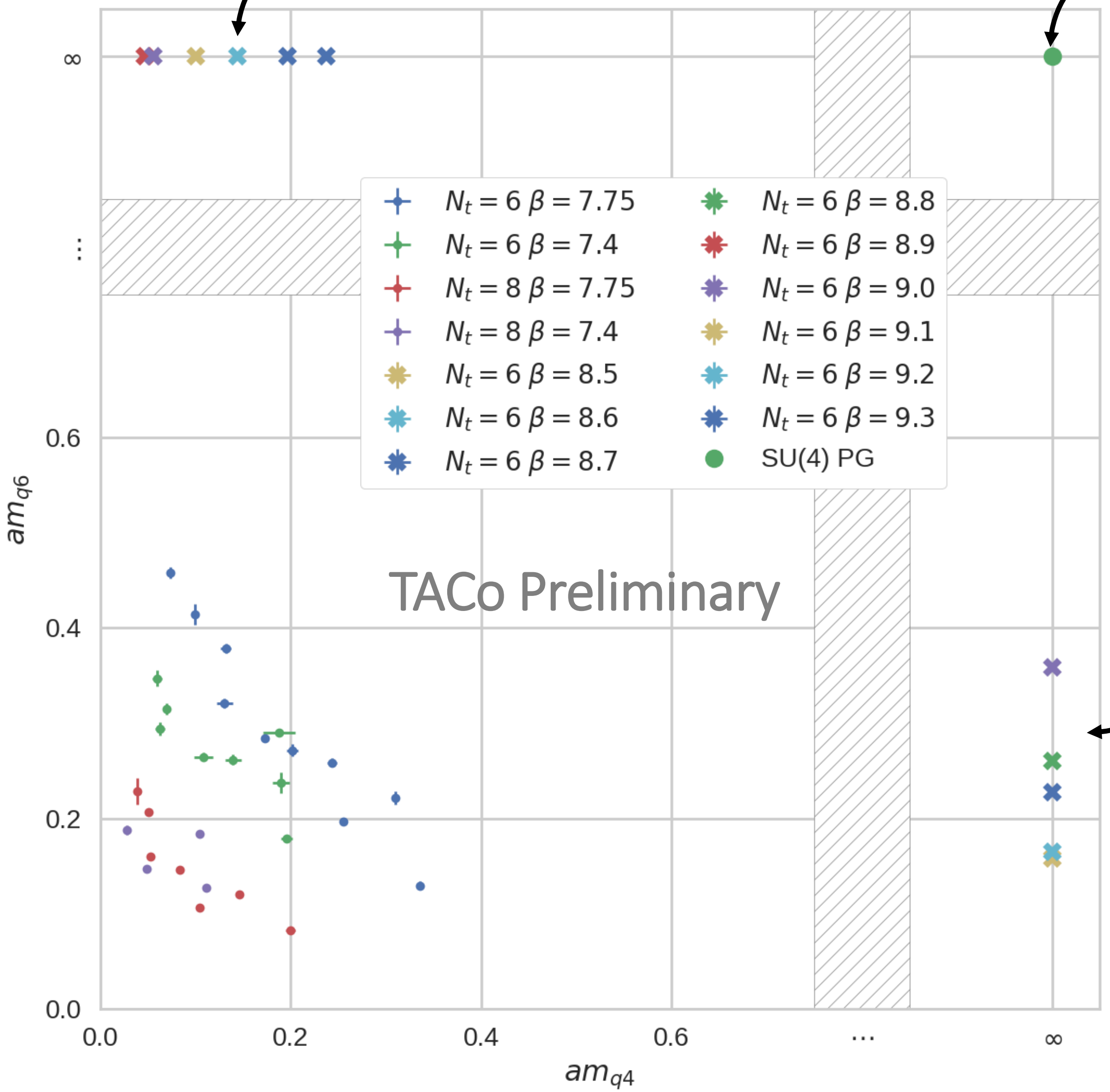
Columbia Plot (Data)

Axial Ward Identity/PCAC quark masses

(Note: Lattice spacing has not been scaled out, so masses are not directly comparable between ensembles.)

F-only limit is continuous, maybe 2nd order

SU(4) PG is 1st order



A_2-only limit is continuous, maybe 2nd order